Conventional and generalized efficiencies of flashing and rocking ratchets: Analytical comparison of high-efficiency limits

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We consider two basic types of Brownian motors which generate directed motion in a periodic asymmetric piecewise-linear potential as a result of random half-period shifts of the potential relief (flashing ratchets) or due to a temporally asymmetric unbiased force applied to the system (rocking ratchets). Analytical relationships have been derived which enable the comparison of the upper limits for the conventional and generalized energy conversion efficiencies in these motors. As found, the increasing amplitude of a sawtooth potential (or the decreasing temperature) makes the conventional efficiency tend to the unity limit faster for a rocking ratchet (in the absence of temporal asymmetry) than for a flashing ratchet. The inverse is true for the generalized efficiency. The potential amplitude being the same, the generalized efficiency is always less than the conventional efficiency. A decreased asymmetry of the potential always results in the reduction of both efficiencies. The temporal asymmetry of an unbiased force has an opposite effect on the conventional and generalized efficiencies: the former rises and the latter drops as the positive signal component becomes shorter in time and larger in amplitude.

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I. INTRODUCTION

Efficiency is a characteristic of paramount importance for a Brownian motor which invokes nonequilibrium fluctuations in asymmetric media to generate directed motion of Brownian particles. A strict thermodynamic definition of this quantity implies that a motor performs the useful work against an external load force F. The conventional efficiency is just specified by the ratio of the useful work performed to the total energy expended [1–3], and it has been extensively and thoroughly studied for various flashing and rocking ratchet models in an effort to ascertain the necessary conditions for maximizing its value [2,4–16]. On the other hand, it is often stated, with good reason [17–21], that nanodevices converting the energy of chemical reactions into the energy of directed motion are remarkable first and foremost not for their ability to do work against external forces but for the very fact of generating directed motion. The directedness originates from the rectification of the nonequilibrium noise which can be characterized by a certain *rectification efficiency*. To explicitly express such an efficiency, one can employ various approaches—for instance, the thermodynamic minimality principle for the useful work performed by the motor [17] or an energy balance based on the averages of the separate terms that make up the Langevin equation without the overdamped approximation for flashing [19] and rocking [20] ratchets. The resulting expression is

$$\eta_r = \frac{F\langle s\rangle + \zeta\langle s\rangle^2}{W_{in}}, \tag{1}$$

where $\langle s\rangle$ is the average velocity of directed motion, ζ is the friction coefficient, and W_{in} is the work expended in unit time. In what follows, we call this quantity the *generalized efficiency* so as to emphasize its more general behavior. Thus, the generalized efficiency differs from the conventional one by an additional addend in the numerator $\zeta\langle s\rangle^2$, which accounts for the work done by friction forces on a particle in unit time. This work is regarded as useful and increases the value of the generalized efficiency as against the conventional efficiency. Importantly, at $F=0$ it is the addend in question that represents the only useful work produced by the motor and thus prevents the efficiency in Eq. (1) from becoming zero.

The sought-for quantities $\langle s\rangle$ and W_{in} are most conveniently found within the overdamped approximation using the Smoluchowski equation as an evolution equation. As an example, if two states $\sigma=\pm$ characterized by the potential energies $U_\sigma(x)$ randomly switch with frequency γ, then a probability $\rho_\sigma(x,t)$ to find the particle in the state σ near the point x at the moment t is specified by the equation

$$\frac{\partial\rho_\sigma(x,t)}{\partial t} = -\frac{\partial}{\partial x}J_\sigma(x,t) - \sigma\gamma[\rho_+(x,t) - \rho_-(x,t)]. \tag{2}$$

Here

$$J_\sigma(x,t) = -De^{-\beta U_\sigma(x)}\frac{\partial}{\partial x}[e^{\beta U_\sigma(x)}\rho_\sigma(x,t)] \tag{3}$$

is the corresponding probability current, $\beta=(k_BT)^{-1}$, k_B is the Boltzmann constant, T is the absolute temperature, and $D=(\beta\zeta)^{-1}$ is the diffusion coefficient. Depending on the model, the potential energies $U_\sigma(x)$ can include a fluctuating periodic contribution $V_\sigma(x)$ (flashing ratchet) or the sum of constant periodic $V(x)$ and fluctuating nonperiodic $-F_{1\sigma}x$ contributions with the fluctuating force $F_{1\sigma}$ (rocking ratchet) as well as the contribution Fx of the load force F. In the

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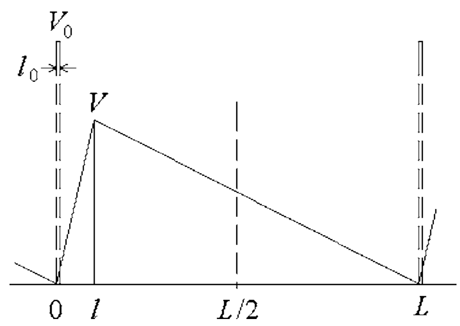

FIG. 1. The potential energy profiles involved in the treatment. At $l \to 0$, a periodic sawtooth potential becomes extremely asymmetric, thus affording the optimum characteristics of the Brownian motor. In this case, the potential on the half-period $(0, L/2)$ is the same as on the half-period $(L/2, L)$, with the vertical displacement $V/2$. The dashed lines designate a narrow and high potential barrier $[l_0 \to 0,\ V_0 \to \infty$ at $V_0 l_0 \to 0$ but $(l_0/L)\exp(\beta V_0) \gg 1]$ which locks the reverse particle motion in a flashing ratchet.

steady state, the total flux $J = J_+(x) + J_-(x)$ is constant and determines the average velocity $\langle s \rangle = JL$ (where L is the potential period), and the quantity W_{in} is given by the expression [6]

$$W_{\mathrm{in}} = \gamma \int_0^L [U_+(x) - U_-(x)][\rho_-(x) - \rho_+(x)]dx. \qquad (4)$$

(A definition of W_{in}, without regard to the overdamped approximation, is presented in [19,20].) For deterministic switches between two states, Eq. (2) without the last addend should be solved separately on each time interval, with the solutions coinciding at the moments of potential switches. In this case, the values $\langle s \rangle$ and W_{in} are found by averaging the corresponding expressions over all the time intervals.

Although a large diversity of highly efficient models of Brownian motors are available in the literature, their efficiencies have been hitherto calculated, as a rule, numerically with some selected parameter sets, whereas an analytical treatment has only been possible for the maximum values of the conventional efficiency regarded as a function of many variables. At the same time, there has been an increasing interest in the generalized efficiency of nanodevices, which calls for a systematic comparative analysis of both efficiency types and elucidation of their upper limits for various Brownian motor models. This problem is addressed in the present study.

It is, therefore, expedient to consider the same potential type in comparing the conventional and generalized efficiencies for the flashing and rocking ratchet models using the overdamped approximation. In this study, we involve a periodic asymmetric piecewise-linear potential (see Fig. 1) which affords, at the extreme degree of asymmetry, the highest efficiencies of both types [12,14]. Section II is devoted to the calculation of the generalized efficiency in such an extremely asymmetric potential for the flashing ratchet; thus, we complement the previous investigation [12] in which the conventional efficiency was calculated under analogous conditions. In Sec. III, the efficiencies of both types are found for the rocking ratchet. The concluding section presents a detailed juxtaposition of the results gained for the two motor models in terms of the conventional and generalized efficiencies.

II. FLASHING RATCHET

An efficiency analysis of flashing ratchets demonstrates that the most promising models are those in which the periodic potential fluctuates via half-period shifts and the potential relief on both half-periods is the same, though differently placed with respect to the energy axis. If this energy shift is compensated by a load force, the conventional efficiency reaches its maximum because a switch between the potentials is not accompanied by the energy losses which arise from the particle transition between the states with different potential reliefs [10,12]. If the reverse motion of the particle is locked by an additional narrow $(l_0 \to 0)$ and high $(V_0 \to \infty)$ barrier, then the fulfillment of the conditions $V_0 l_0 \to 0$ and $(l_0/L)\exp(\beta V_0) \gg 1$ implies that the conventional efficiency approaches unity as

$$\eta \to \eta_\infty - \sqrt{L/2l_0}\exp(-\beta V_0/2), \qquad (5)$$

with $\eta_\infty = 1$ [10]. Importantly, the limiting expression obtained is independent of the potential shape and holds in a wide range of potential switching frequencies γ. Strictly speaking, the regularity revealed is also valid at $\gamma \to \infty$, since a periodic potential with the energy-shifted identical reliefs on two half-periods can exist only provided that there are discontinuities in its profile at the points $x = L/2$ and L. Assuming that the jump of the potential by the value V occurs within a region of a nonzero width l, the frequency range is restricted by the condition $\gamma \ll D(\beta V/l)^2$.

With the rising barrier height V_0, the generalized efficiency displays the same limiting behavior, with the only difference that the value η_∞ is under unity and depends on the model parameters. The smaller efficiency value results from the absence of a load force: the energy shift between the identical potential reliefs on both half-periods cannot be compensated and hence certain energy losses arise. Nonetheless, the value of η_∞ goes to unity if the condition $\Gamma \equiv \gamma L^2/D \gg 1$ is met and provided that the characteristic height of the potential relief V satisfies the inequality $v^2 \ll \Gamma$, where $v = \beta V$. It follows from exact analytical relations [10] that the above inference holds true for an arbitrary shape of the potential relief on a half-period. In the particular case of an extremely asymmetric sawtooth potential (see Fig. 1 with $l \to 0$), the expression for η_∞ appears as

$$\eta_\infty = \left\{1 + \frac{4\Delta[\cosh(v/4) + \cosh \Delta]}{\Gamma \sinh \Delta}\right\}^{-1}, \qquad (6)$$

where $\Delta = \sqrt{v^2 + 8\Gamma}/4$. At $\Gamma \to \infty$, the value of η_∞ tends to unity as $1 - \sqrt{8/\Gamma}$.

In the absence of an additional barrier, the reverse particle motion is locked due to the peculiar shape of the potential relief. For an extremely asymmetric sawtooth potential, the locking is governed by the parameter V. The conventional efficiency dependent on V is a function of both the potential switching frequency γ and the load force F. The maximum of the function η with respect to the variables γ and F is given by the relation [12]

$$\eta^{\max} \approx 1 - 2\frac{\ln v}{v}, \quad v \gg 1, \tag{7}$$

at

$$\Gamma \approx 4\frac{\ln v}{v}, \quad F \approx \frac{V}{L}\eta^{\max}, \quad l = 0. \tag{8}$$

It is noteworthy that the maximum efficiency is reached at so large load forces that the corresponding average velocity is small and equal to $\langle s\rangle \to (2/\beta\zeta L)(\ln v/v)$. As a consequence, the values of the conventional and generalized efficiencies differ by a small value of the order $\zeta\langle s\rangle/F \approx (2/v^2)\ln v$ and practically coincide at their maximum specified by Eq. (7).

From the previously reported relationships [12], one can also define the quantity W_{in} needed to calculate the generalized efficiency (1). Setting $F=0$ and $l=0$, we write the exact expression

$$\eta_{\mathrm{r}} = \frac{2A^2}{\Gamma\sinh(v/2)[\exp(v/2)-1](Z_1B - Z_2A)}, \tag{9}$$

where

$$A = [\Psi_0\exp(v/2) - \Psi_L][\cosh(v/2) - 1],$$

$$B = \Psi_0\exp(v/2)\cosh(v/2) + \Psi_L,$$

$$Z_1 = \frac{16}{v^2}\sinh^2(v/4), \quad Z_2 = \frac{4}{v^2}[\exp(v/2) - 1 - (v/2)],$$

$$\Psi_{0,L} = \frac{1}{4\sinh\Delta}\{4\Delta[\exp(\mp v/4) + \cosh\Delta] \mp v\sinh\Delta\},$$

$$\Delta = \frac{1}{4}\sqrt{v^2 + 8\Gamma}. \tag{10}$$

At v, $\Gamma \gg 1$, Eqs. (9) and (10) are simplified, so that the desired efficiency takes the following approximate form:

$$\eta_{\mathrm{r}} \approx \frac{2v(z-1)}{v(z^2-1) + 2(z+1)}, \quad z = \sqrt{1 + 8\Gamma/v^2}. \tag{11}$$

The function $\eta_{\mathrm{r}}(z)$ assumes the maximum value

$$\eta_{\mathrm{r}}^{\max} = \frac{v}{(\sqrt{2} + \sqrt{v+2})^2} \tag{12}$$

at

$$\Gamma = \frac{1}{2}v^2\left(\sqrt{\frac{2}{v+2}} + \frac{2}{v+2}\right). \tag{13}$$

Interestingly, the maximum value of unity is reached for the conventional efficiency at small potential switching frequencies and for the generalized efficiency at large frequencies. In the limit $v \to \infty$, we obtain the expression $\eta_{\mathrm{r}}^{\max} \to 1 - 2\sqrt{2/v}$ which goes to unity slower than formula (7).

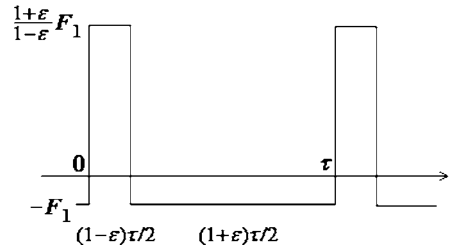

FIG. 2. The temporally asymmetric unbiased force giving rise to directed motion of Brownian particles in a rocking ratchet.

III. ROCKING RATCHET

Rocking ratchets generate the directed motion of a Brownian particle not only due to the asymmetry of a periodic potential but also by means of a temporally asymmetric unbiased force [22]:

$$F_1(t) = \begin{cases} \kappa F_1, & n\tau \le t < n\tau + (1-\varepsilon)\tau/2, \\ -F_1, & n\tau + (1-\varepsilon)\tau/2 < t \le (n+1)\tau, \end{cases} \tag{14}$$

where the parameters ε and $\kappa \equiv (1+\varepsilon)/(1-\varepsilon)$ signify the temporal asymmetry $(0 \le \varepsilon < 1)$, τ is the period of the driving force $F_1(t)$, and n is an integer (see Fig. 2). Expression (14) leads the average value of the force $F_1(t)$ to reduce to zero. In addition to this force, the load force F should be introduced for the conventional efficiency to be calculated. Then the slopes of the linear sections in the sawtooth potential, $0 \le x < l$ and $l \le x < L$ (see Fig. 1), become $f_+ = V/l - f$ and $f_- = -V/(L-l) - f$, respectively, where the quantity f defines the total external force $\kappa F_1 - F$ and $-F_1 - F$ acting on the particle during the respective time intervals $(1-\varepsilon)\tau/2$ and $(1+\varepsilon)\tau/2$. In the adiabatic approximation, one can assume that such time intervals are sufficient for equilibrium to be established. Then the flux $J_0(f)$ within either interval appears as follows [14,23]:

$$\beta D[J_0(f)]^{-1} = -\frac{l}{f_+} - \frac{L-l}{f_-} + \beta^{-1}\left\{\frac{e^{\beta f_-(L-l)} - 1}{f_-^2} - \frac{e^{-\beta f_+ l} - 1}{f_-^2} + \frac{1}{e^{-\beta f_+ l} - e^{\beta f_-(L-l)}}\left[\frac{e^{\beta f_-(L-l)} - 1}{f_-} - \frac{e^{-\beta f_+ l} - 1}{f_-}\right]^2\right\}, \tag{15}$$

whereas the average velocity of the Brownian particle, $\langle s\rangle$, and the energy expended in unit time W_{in} are defined by the relations [15]

$$\langle s\rangle = \frac{1}{2}L(1-\varepsilon)[J_0(\kappa F_1 - F) + \kappa J_0(-F_1 - F)],$$

$$W_{\mathrm{in}} = \frac{1}{2}F_1(1+\varepsilon)[J_0(\kappa F_1 - F) - J_0(-F_1 - F)]. \tag{16}$$

The above formulas enable numerical calculation of the conventional and generalized efficiencies in relation to various parameters of the model [15,24]. At the same time, the analysis of the maximum possible efficiencies calls for sufficiently simple analytical expressions which enable a convenient search for extremum values. First we consider the low-temperature limit which is known to afford, at $\varepsilon=0$, the maximum conventional efficiency, with its value depending on the spatial asymmetry parameter $\lambda \equiv l/L$ [14]. In this case, the average velocity $\langle s\rangle$ assumes negative values, in contrast to flashing ratchets for which $\langle s\rangle$ is positive. The efficiency upper limits are obtainable from the evident condition $|J_0(\kappa F_1-F)| \gg |J_0(-F_1-F)|$ valid at $f_+=V/l-\kappa F_1+F \to 0$—i.e., at $F\to \kappa F_1 - V/l$. Thus, the upper limit of the conventional efficiency is specified by the expression

$$\eta^* = 1 - \frac{1+\kappa}{\kappa}\lambda. \tag{17}$$

For the generalized efficiency, one should involve the low-temperature asymptotic of the flux $J_0(\kappa F_1)$ to obtain

$$\eta_r^* = \frac{1}{\kappa}\frac{\kappa(1-\lambda)-\lambda}{\kappa(1-\lambda)+1-2\lambda}. \tag{18}$$

Note that $\langle s\rangle$ and both efficiencies become zero only if the model has neither spatial $(\lambda=1/2)$ nor temporal $(\kappa=1)$ asymmetry. Figure 3 demonstrates the upper limits of both efficiencies versus the spatial asymmetry parameter λ at the varied values of the temporal asymmetry parameter κ. It is seen that the highest efficiencies are always reached at the extremely asymmetric potential $(\lambda=0)$. In this case, we have $\eta^*=1$ at any values of κ, whereas $\eta_r^*=(\kappa+1)^{-1}$ (if there is no temporal asymmetry, then $\eta_r^*=1/2$).

Restricting our consideration to the case of the extremely asymmetric potential, we now derive the relations which describe the tendency of the efficiencies to their upper limits in the low-temperature region. Writing the asymptotic expressions for the fluxes $J_0(\kappa F_1-F)$ and $J_0(-F_1-F)$ at $v=\beta V \gg 1$ and substituting α for F with regard to the relation $F=\kappa F_1-\alpha(V/l)$, we find the conventional efficiency at $l\to 0$:

$$\eta(F_1,\alpha) \approx 1 - \alpha\frac{V}{\kappa F_1 L} - \frac{2(\kappa+1)^2}{(\alpha-1)v}(\beta F_1 L)^2 \exp(-v). \tag{19}$$

The maximum of this function of two variables is found as

$$\eta^{\max} \approx 1 - 3(1+\kappa^{-1})^{2/3}(2v)^{1/3}\exp(-v/3). \tag{20}$$

The generalized efficiency is given by the following function:

$$\eta_r(F_1) \approx \frac{1}{\kappa+1}\left[1 - \frac{V}{\kappa F_1 L} - \frac{2\kappa+1}{\kappa}\beta F_1 L \exp(-v)\right], \tag{21}$$

with the maximum

$$\eta_r^{\max} \approx \frac{1}{\kappa+1}\left[1 - \frac{2}{\kappa}\sqrt{(2\kappa+1)v}\exp(-v/2)\right]. \tag{22}$$

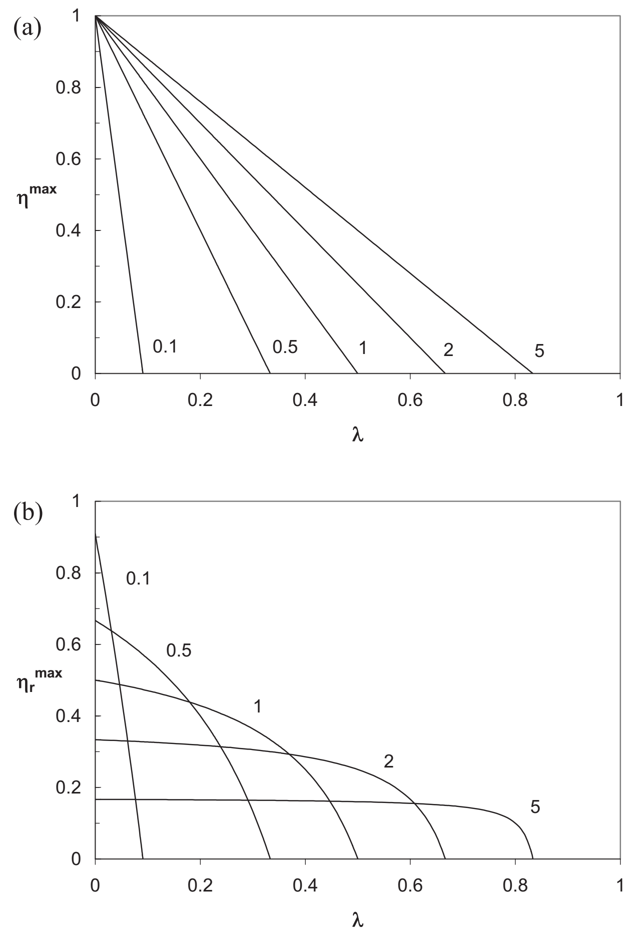

FIG. 3. The upper limits of the conventional (a) and generalized (b) efficiencies for a rocking ratchet plotted versus the spatial asymmetry parameter $\lambda \equiv l/L$ (see Fig. 1) at varied values of the temporal asymmetry parameter $\kappa \equiv (1+\varepsilon)/(1-\varepsilon)$ (see Fig. 2).

IV. CONCLUSIONS

We have considered two basic types of Brownian motors which generate directed motion in a periodic asymmetric piecewise-linear potential as a result of random half-period shifts of the potential relief (flashing ratchets) or due to a temporally asymmetric unbiased force applied to the system (rocking ratchets). We are concerned with the upper limits of the generalized energy conversion efficiency represented by relation (1) with and without an external load force F. The generalized efficiency differs from the conventional one in that the numerator in Eq. (1) contains, besides the useful work $F\langle s\rangle$, an additional addend $\zeta\langle s\rangle^2$, which accounts for the work done by friction forces on a particle in unit time. The upper limit of the generalized efficiency at $F\neq 0$ is realized at so small values of the average velocity $\langle s\rangle$ that $\zeta\langle s\rangle^2 \ll F\langle s\rangle$ and the generalized efficiency is much the same as the conventional efficiency. Thus, the analysis of the maximum efficiency suggests the conventional or generalized efficiency according to whether the load force is present or not. The former is significant if the motor functioning implies the work against external forces, whereas the latter characterizes the motors generating directed motion as such

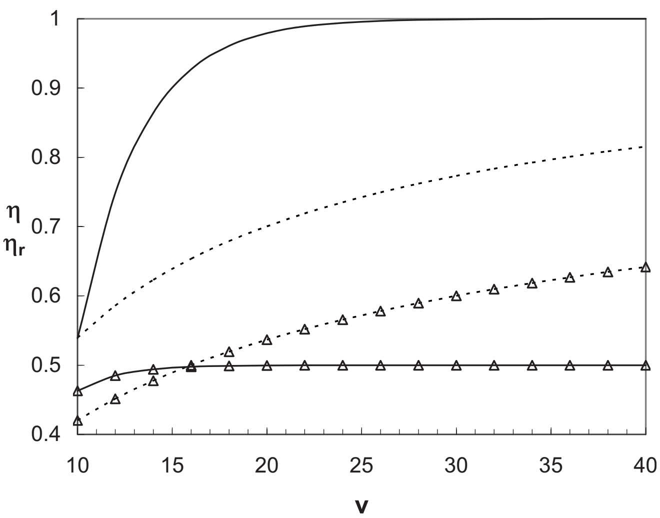

FIG. 4. The conventional (lines without markers) and generalized (lines with triangles) efficiencies for a rocking (solid lines) and flashing (dashed lines) ratchets plotted versus the ratio V/k_BT, where V is the amplitude of the extremely asymmetric sawtooth potential. The plots represent Eqs. (7) and (12) for the flashing ratchet and Eqs. (20) and (22) with $\kappa=1$ and hence $\varepsilon=0$ (i.e., without the temporal asymmetry) for the rocking ratchet.

and aimed at the maximum average velocity. The behavior of the two efficiencies has many features in common: both of them increase with the potential amplitude and asymmetry—i.e., with the factors which are just responsible for the origination of directed motion. At the same time, some distinctions between them are revealed which arise from the method of directed motion generation and from the peculiarities of the definition of useful work.

For a flashing ratchet, the most efficient energy conversion occurs if a reverse particle flux is locked by an additional high and narrow barrier. In this case, the limiting low-temperature behavior of the conventional and generalized efficiencies is accounted for by Eqs. (5) and (6); i.e., their maximum values are reached most rapidly. In contrast, if an additional high barrier were involved in the potential of a rocking ratchet, it would result in a very low speed of the particle flux. Therefore, the further comparative analysis of the high-efficiency limits was performed for a periodic asymmetric piecewise-linear potential without additional barriers. Analytical relations (7), (12), (20), and (22) dictate how rapidly the maximum values of the conventional and generalized efficiencies approach their upper limits with decreasing temperature (the corresponding plots are presented in Fig. 4). As seen, the increasing amplitude of a sawtooth potential (or the decreasing temperature) makes the conventional efficiency tend to the unity limit faster for a rocking ratchet (in the absence of temporal asymmetry) than for a flashing ratchet. The inverse is true for the generalized efficiency. The potential amplitude being the same, the generalized efficiency is always less than the conventional efficiency. A decreased asymmetry of the potential always results in the reduction of both efficiencies. The temporal asymmetry of an unbiased force has an opposite effect on the conventional and generalized efficiencies: the former rises and the latter drops as the positive signal component becomes shorter in time and larger in amplitude (see Fig. 3).

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